

# Dilute Fermi gas in quasi-one-dimensional traps: From weakly interacting fermions via hard core bosons to weakly interacting Bose gas

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We study equilibrium properties of a cold two-component Fermi gas confined in a quasi-one-dimensional trap of the transverse size  $l_\perp$ . In the dilute limit ( $nl_\perp \ll 1$ , where  $n$  is the 1D density) the problem is exactly solvable for an arbitrary 3D fermionic scattering length  $a_F$ . When  $l_\perp/a_F$  goes from  $-\infty$  to  $+\infty$ , the system successively passes three regimes: weakly interacting Fermi gas, hard core Bose gas and weakly coupled Bose gas. The regimes are separated by two crossovers at  $a_F \sim \pm nl_\perp^2$ . In conclusion we discuss experimental implications of these results.

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Trapped cold atomic gases [1] offer a unique possibility to create experimentally various many-body systems that for a long time have been considered as purely theoretical models. In this respect the study of quasi-one-dimensional (quasi-1D) systems is especially promising since 1D many-body problems are frequently exactly solvable [2]. Currently quasi-1D cold atomic Bose gases are indeed extensively studied both experimentally [3] and theoretically [4, 5, 6]. The behavior of two-component Fermi systems is expected to be more diverse due to one more "degree of freedom" related to the formation of two-particle composite bosons (dimers). In quasi-3D traps both Bose-Einstein condensation (BEC) of dimers [7] and the regime of a crossover from Bardeen-Cooper-Schrieffer (BCS) superfluidity to molecular BEC [8] have been recently reproduced experimentally. While theoretical studies of BCS-BEC crossover in 3D (or quasi-3D) systems have a long history (see Refs. 9, 10, 11, 12, 13, 14 and references therein), a similar problem for a strongly anisotropic confinement has not been addressed up to now. In this paper we present a complete theory of "BCS-BEC" transformation from weakly coupled Fermi gas to Bose gas of diatomic molecules in quasi-1D traps.

We consider a dilute two-component Fermi gas confined in the  $(x, y)$  plane by a harmonic potential with characteristic scale  $l_\perp = (m\omega_\perp)^{-1/2}$ . In the present context the diluteness means that  $nl_\perp \ll 1$ , where  $n$  is the density averaged over transverse directions. As usual, assuming that the interaction potential  $V(\mathbf{r})$  is of short range  $R_W \ll l_\perp$ , we employ the standard pseudopotential approximation:  $V(\mathbf{r}) = \frac{4\pi a_F}{m} \delta(\mathbf{r}) \frac{\partial}{\partial r} r$ , where  $a_F$  is the 3D scattering length. Parameter  $l_\perp/a_F$  ranges from  $-\infty$  to  $+\infty$ , which corresponds to an attractive interaction and includes the unitarity limit ( $a_F = \infty$ ). We will show that in the whole range of  $a_F$  this system is described by exactly solvable models. At small negative  $a_F$  ( $l_\perp/a_F \ll -1$ ) it behaves according to Gaudin's solution [15] for an attractive 1D Fermi gas. At  $a_F \sim -nl_\perp^2$  the Gaudin's Fermi gas transforms to Girardeau-Tonks gas of hard core bosons [16, 17], which, at positive  $a_F > nl_\perp^2$ , smoothly matches

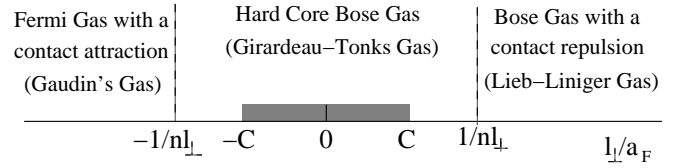


FIG. 1: Phase diagram of quasi-1D Fermi gas.

the regime of Lieb-Liniger Bose gas [18] with weak repulsion. A schematic phase diagram is shown in Fig. 1. Two crossovers in Fig. 1 reflect two different physical phenomena. The left crossover corresponds to the transformation from strongly overlapped BCS-like pairs to well defined composite bosons, which is roughly similar to the common BCS-BEC crossover in 3D systems. The right crossover is inherent for quasi-1D geometry. It is related to a change of boson-boson interaction due to a "dimensional transformation" from quasi-1D to 3D character of composite bosons.

Many-body physics of an attractive Fermi gas is closely related to the solution of a two-body problem. The two-body scattering problem in an axially symmetric harmonic trap has been already addressed by several authors [4, 5, 19]. Below we briefly discuss properties of a two-body bound state that is a localized solution to Schrödinger equation for the relative motion. The energy of this state should have a form  $E_0 = \omega_\perp - \varepsilon_0$ , where  $\varepsilon_0 > 0$  is the binding energy. In the pseudopotential approximation the bound state wave function,  $\chi_0(\mathbf{r})$ , is expressed in terms of Green's function,  $G(E, \mathbf{r}, \mathbf{r}')$ , of a cylindrical harmonic oscillator with the mass  $m/2$ :

$$\chi_0(\mathbf{r}) = AG_0(\mathbf{r}, 0), \quad (1)$$

Here  $G_0(\mathbf{r}, 0) \equiv G(E_0, \mathbf{r}, 0)$ , and  $A$  is the normalization constant which is given by the expression

$$A^2 = \frac{32\pi}{\sqrt{2}m^2l_\perp} \left[ \zeta \left( \frac{3}{2}, \frac{\varepsilon_0}{2\omega_\perp} \right) \right]^{-1} \quad (2)$$

where  $\zeta(z, \Omega)$  is the zeta function [20]. The binding energy  $\varepsilon_0$  is a solution to the following transcendental equation

$$\frac{m}{4\pi a_F} = \left[ \frac{\partial}{\partial r} r G_0(\mathbf{r}, 0) \right]_{r=0}. \quad (3)$$

Equation (3), which defines a pole in the two-body scattering amplitude, can be reduced to the form [5]

$$l_\perp/a_F = -\zeta(1/2, \varepsilon_0/2\omega_\perp)/\sqrt{2} \quad (4)$$

Right hand side in Eq. (4) is a monotonically increasing function of  $\Omega = \varepsilon_0/2\omega_\perp$ . It diverges as  $-1/\sqrt{2\Omega}$  at  $\Omega \ll 1$ , crosses zero at  $\Omega \approx 0.3$ , and goes to  $+\infty$  as  $\sqrt{2\Omega}$  at  $\Omega \gg 1$ . This behavior translates to the monotonic dependence of the binding energy  $\varepsilon_0$  on  $a_F$ . For small negative  $a_F$  ( $l_\perp/a_F \ll -1$ ) we get the result  $\varepsilon_0 \approx a_F^2/ml_\perp^4$ . In this regime  $\varepsilon_0 \ll \omega_\perp$  which means that the transverse motion of particles is confined to the lowest state of spatial quantization. The bound state is strongly anisotropic with the axial size  $\sim 1/\sqrt{m\varepsilon_0} \gg l_\perp$ . In the unitarity limit,  $l_\perp/a_F = 0$ , the bound state wave function becomes almost spherically symmetric, while the energy takes a universal form  $\varepsilon_0 = 0.6\omega_\perp$ . In the regime of strong attraction ( $l_\perp/a_F \gg 1$ ) the binding energy approaches the usual 3D expression:  $\varepsilon_0 \approx 1/ma_F^2$ .

We also introduce an effective size of the bound state  $a_0 = 1/\sqrt{m\varepsilon_0}$ , which has the following behavior in the characteristic regions of  $l_\perp/a_F$

$$a_0 \approx l_\perp^2/|a_F| \gg l_\perp, \quad l_\perp/a_F \ll -1, \quad (5)$$

$$a_0 \approx 1.29l_\perp, \quad l_\perp/a_F = 0, \quad (6)$$

$$a_0 \approx a_F \ll l_\perp, \quad l_\perp/a_F \gg 1. \quad (7)$$

Let us turn to the many-body problem in the limit  $nl_\perp \ll 1$ . We start from the regime of weak attraction:  $a_F < 0$  and  $|a_F| \ll l_\perp$  (the region to the left of the shaded region in the phase diagram of Fig. 1). In this case the transverse motion of each Fermi particle is confined to the lowest oscillator's state. Therefore an effective 1D many-body Hamiltonian is obtained from the original one by the simple averaging over transverse directions

$$H_F = \int dz \left[ \sum_{j=1}^2 -\psi_j^\dagger \frac{\partial^2}{2m} \psi_j - g_F \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1 \right], \quad (8)$$

where  $\psi_j(z)$  are the Fermi field operators, and the 1D fermionic coupling constant takes the form

$$g_F = 2|a_F|/ml_\perp^2. \quad (9)$$

Solving the 1D two-particle problem with the point attractive interaction of Eq. (9) we find the bound state energy  $\varepsilon_0 = mg_F^2/4 = a_F^2/ml_\perp^4$ , which recovers the corresponding solution to Eq. (4).

The problem of 1D spin-1/2 fermions with an attractive interaction, Eq. (8), is exactly solvable [15]. The

ground state energy  $E_F$  is given by the set of Gaudin's integral equations [15]

$$E_F = -\frac{1}{8}mg_F^2n + 2 \int_{-K_0}^{K_0} \frac{k^2}{2m} f(k) \frac{dk}{2\pi}, \quad (10)$$

$$f(k) = 2 - \int_{-K_0}^{K_0} \frac{2mg_F}{(mg_F)^2 + (k-k')^2} f(k') \frac{dk'}{2\pi}, \quad (11)$$

$$n = 2 \int_{-K_0}^{K_0} f(k) \frac{dk}{2\pi}. \quad (12)$$

The solution of Eqs. (10)-(12) is governed by dimensionless parameter  $\gamma_G = mg_F/n \sim 1/na_0 \sim |a_F|/nl_\perp$ . Since  $\gamma_G$  is a ratio of two small parameters ( $|a_F|/l_\perp$  and  $nl_\perp$  respectively) it can take any value in the region of applicability of Eqs. (8)-(12).

In the weak coupling limit,  $\gamma_G \ll 1$  ( $l_\perp/a_F \ll -1/nl_\perp$ ), we get a weakly interacting two-component 1D Fermi gas with the energy  $E_F^0 = \pi^2 n^3/24m$ . To reveal the physics behind the strong coupling limit,  $\gamma_G \gg 1$  ( $l_\perp/a_F \ll -1/nl_\perp$ ), it is instructive to rescale all momenta,  $k \rightarrow p/2$ , in Eqs. (10)-(12) and rewrite them as follows

$$E_F = -\varepsilon_0 \frac{n}{2} + \int_{-P_0}^{P_0} \frac{p^2}{2M} \tilde{f}(p) \frac{dp}{2\pi}, \quad (13)$$

$$\tilde{f}(p) = 1 - \int_{-P_0}^{P_0} \frac{2Mg_F}{(Mg_F)^2 + (p-p')^2} \tilde{f}(p') \frac{dp'}{2\pi}, \quad (14)$$

$$\frac{n}{2} = \int_{-P_0}^{P_0} \tilde{f}(p) \frac{dp}{2\pi}, \quad (15)$$

where  $\tilde{f}(p) = 2f(p/2)$ , and  $M = 2m$ . Obviously, the first term in Eq. (13) is the energy of  $n/2$  noninteracting bound states (composite bosons). The rest of the set of Eqs. (13)-(15) coincides (except for the sign of the integral term in Eq. (14)) with a set of Lieb-Liniger equations [18] for a gas of interacting Bose particles with the mass  $M$  and the density  $n/2$ . This analogy was already noted in the early paper by Gaudin [15]. In the strong coupling limit the integral term in Eq. (14) (with a “wrong” sign) is irrelevant. Hence in this regime the system behaves as a gas of impenetrable composite bosons (Girardeau-Tonks gas). The strong repulsion is a direct consequence of Fermi statistics and 1D kinematics – the composite particles can not go around while the Pauli principle forbids them to penetrate each other. The energy in this regime is given by the formula for a one-component Fermi gas of the density  $n/2$ :  $E_F^\infty = -\varepsilon_0 n/2 + \pi^2 (n/2)^3/6M$ . A crossover from the weakly coupled Fermi gas to the Tonks gas is located at  $l_\perp/a_F \sim -1/nl_\perp$ . The description of our system in terms of Gaudin's Hamiltonian, Eq. (8), with  $g_F$  of Eq. (9) is valid in the region  $l_\perp/a_F < -C$ , where constant  $C$  satisfies the condition  $1/nl_\perp \gg C \gg 1$ .

Another asymptotic regime corresponds to  $a_F > 0$  and  $a_F \ll l_\perp$  (the region to the right of the shaded region in Fig. 1). This is the regime of small 3D composite

bosons with the size  $a_0 = a_F$  (see Eq. (7)), the binding energy  $\varepsilon_0 = 1/ma_F^2$  and 3D scattering length  $a_B \approx 0.6a_F$  [21, 22]. Since  $nl_\perp \ll 1$ , the transverse center-of-mass motion of dimers corresponds to the lowest harmonic oscillator state. Hence the effective 1D bosonic Hamiltonian reduces to the form

$$H_B = -\varepsilon_0 \frac{n}{2} + \int dz \left[ -\varphi^\dagger \frac{\partial_z^2}{2M} \varphi + \frac{1}{2} g_B \varphi^\dagger \varphi^\dagger \varphi \varphi \right] \quad (16)$$

with the following 1D bosonic coupling constant

$$g_B = 2a_B/ml_\perp^2 \approx 1.2a_F/ml_\perp^2. \quad (17)$$

The energy of  $n/2$  bosons with a repulsive contact interaction is given by the exact solution due to Lieb and Liniger [18] (Lieb-Liniger integral equations are equivalent to Eqs. (10)-(12) with the replacement  $g_F \rightarrow -g_B$ ).

A dimensionless parameter that governs the behavior of the system in this regime is  $\gamma_{LL} = Mg_B/n \sim a_F/nl_\perp^2$ . In the region  $l_\perp/a_F \gg 1/nl_\perp$  (which corresponds to  $\gamma_{LL} \ll 1$ ) we have a weakly coupled Bose gas with the energy  $E_B^0 = -\varepsilon_0 n/2 + \frac{1}{2} g_B (n/2)^2$  [18]. In the opposite limit  $l_\perp/a_F \ll 1/nl_\perp$  ( $\gamma_{LL} \gg 1$ ) we again recover the hard core 1D bosons, and thus get the energy  $E_B^\infty = -\varepsilon_0 n/2 + \pi^2 (n/2)^3/6M$ . A crossover between these two regimes corresponds to  $l_\perp/a_F \sim 1/nl_\perp$ .

Inside the shaded region in Fig. 1 both above asymptotic approaches (Eqs. (8)-(9) and Eqs. (16)-(17)) are not applicable. In this region the relative motion two particles is neither 1D (as assumed in Eqs. (8)-(9)) nor 3D (which is the condition for applicability of Eq. (17)). We have, however, seen that on either side of the shaded region the system behaves a gas of hard core composite bosons. Below we prove that this model is also valid everywhere inside. The main point is that for all  $l_\perp/a_F \gg -1/nl_\perp$ , which also covers the questionable region, the bound state's size  $a_0$ , Eqs. (5)-(7), is much smaller than mean interparticle distance  $1/n$ . This allows us to apply the functional integral approach that has been originally developed to describe the molecular limit of BCS-BEC crossover in homogeneous systems [9, 10, 11]. The generalization of this theory to arbitrary quasi-3D trapped systems is given in Ref. 22. Since in harmonic traps the center-of-mass and the relative motions are decoupled, the general formalism of Ref. 22 is directly applicable to the present quasi-1D problem. The result of this approach is quite simple and physically appealing – to the leading order in  $a_0 n \ll 1$  the system is still described by the bosonic Hamiltonian of Eq. (16), but  $g_B$  in general has a more complicated form. In the present context it is enough to consider a Born approximation to the coupling constant  $g_B$ , which corresponds to an “exchange” process shown in Fig. 2 [13, 22]. An analytic expression for this diagram takes a form

$$g_B = \Lambda_0^4 \int t_B(\rho_1, \rho_2, \rho_3, \rho_4) \prod_{i=1}^4 \Phi_{0,0}(\rho_i) d^2 \rho_i, \quad (18)$$

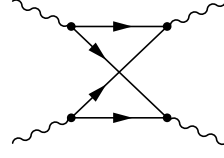


FIG. 2: Diagram for the Born scattering amplitude of two composite bosons.

where  $\rho = (x, y)$  is the transverse coordinate,  $\Phi_{0,0}(\rho) = l_\perp^{-1} \sqrt{2/\pi} e^{-(\rho/l_\perp)^2}$  is the wave function for the transverse motion of the center-of-mass, and  $\Lambda_0 = \int V(\mathbf{r}) \chi_0(\mathbf{r}) d^3 \mathbf{r} \equiv A$  (see Eqs. (1), (2)) is the boson-fermion “vertex” [22]. The four-point function  $t_B$  is defined as follows

$$t_B = \int \frac{d\omega dk}{(2\pi)^2} \mathcal{G}_{\omega,k}(\rho_1, \rho_2) \mathcal{G}_{\omega,k}^*(\rho_2, \rho_3) \times \mathcal{G}_{\omega,k}(\rho_3, \rho_4) \mathcal{G}_{\omega,k}^*(\rho_4, \rho_1), \quad (19)$$

Here  $\mathcal{G}_{\omega,k}(\rho, \rho')$  is the one particle Green's function for a harmonically confined Fermi gas with the chemical potential  $\mu = \omega_\perp - \varepsilon_0/2$

$$\mathcal{G}_{\omega,k} = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{\phi_{n,m}(\rho) \phi_{n,m}^*(\rho')}{i\omega - \frac{k^2}{2m} - \omega_\perp(2n + |m|) - \frac{\varepsilon_0}{2}} \quad (20)$$

where  $\phi_{n,m}(\rho)$  are the eigen functions of a 2D harmonic oscillator with frequency  $\omega_\perp$ . Reducing all integrals in Eqs. (18), (19) to a dimensionless form we find that  $g_B$  has the following general structure

$$g_B = \tilde{g}(\varepsilon_0/\omega_\perp)/ml_\perp \quad (21)$$

Dimensionless function  $\tilde{g}(x)$  increases as  $6\sqrt{x}$  at  $x \ll 1$ , reaches a maximum at  $x \sim 1$ , and goes to zero as  $4/\sqrt{x}$  at  $x \gg 1$ . Using the known dependence of  $\varepsilon_0$  on  $l_\perp/a_F$  we get the required function  $g_B(a_F)$  in all characteristic regimes:  $g_B \sim |a_F|/ml_\perp^2$  if  $l_\perp/|a_F| \gg 1$ , and  $g_B \sim 1/ml_\perp$ , if  $l_\perp/|a_F| < 1$  [23]. Renormalization of the Born scattering amplitude does not alter this general behavior since it can change only numerical coefficients in the above formulas. Thus, everywhere in the region  $l_\perp/|a_F| \ll 1/nl_\perp$  the strong coupling condition  $mg_B/n \gg 1$  is fulfilled. This proves the correctness of the complete phase diagram in Fig. 1.

The above formal results lead to the following physical picture of the quasi-1D “BCS-BEC” transformation that consists of two well defined steps. The first step is the “Fermi-Bose” crossover at  $l_\perp/a_F \sim -1/nl_\perp$ . On the left of this point we have a “BCS”-like state with strongly overlapped pairs, while everywhere on the right the system is composed of well defined composite bosons. The further physical changes in the system are related to the evolution of an effective boson-boson interaction. In the region  $-1/nl_\perp < l_\perp/a_F < C$  (from the left crossover to the right end of the shaded region in Fig. 1) the bosons

are of quasi-1D form. Their transverse size equals to the transverse size  $l_\perp$  of the confining potential. Therefore two colliding bosons can not avoid each other while their mutual penetration is forbidden by Pauli principle. As a result we have Girardeau-Tonks regime. In the region  $C > l_\perp/a_F > 1/nl_\perp$  the composite bosons take the common 3D form with radius  $a_0 \approx a_F$ . However the transverse confinement of their center-of-mass motion is still strong – the system remains in the hard core gas regime. It should be outlined that the microscopic reason for the Tonks gas regime is quite different on opposite sides of the shaded region. Finally when the size of bosons becomes small enough ( $a_0 \approx a_F < nl_\perp^2$ ) the system enters the regime of weakly interacting Bose gas. It is worse mentioning that in contrast to 3D case nothing special happens in the unitarity limit  $a_F = \infty$  that is simply the middle of the Girardeau-Tonks regime. In fact this regime is a quasi-1D realization of a universal behavior when all thermodynamic characteristics are independent of the fermionic scattering length. An important feature of quasi-1D systems is that the universal behavior extends to a wide region around the point  $a_F = \infty$ .

Experimentally quasi-1D atomic gases are normally produced in highly elongated needle-shaped traps [3] while the scattering length  $a_F$  can be tuned using the Feshbach resonance technique [7, 8]. In such systems the double-crossover structure of the phase diagram can be observed via changes of the density profile  $n(z)$ . If the axial confinement (with a scale  $l_z \gg l_\perp$ ) is semiclassical we can calculate  $n(z)$  using the local density approximation. In the regime of weakly interacting Gaudin's gas we get the result  $n_G(z) = (2/\pi l_z) \sqrt{N - z^2/l_z^2}$ , which corresponds to noninteracting fermions (here  $N$  is the total number of particles). The density distribution in the Girardeau-Tonks regime is given by the similar expression  $n_{GT}(z) = (2\sqrt{2}/\pi l_z) \sqrt{N - 2z^2/l_z^2} = \sqrt{2}n_G(\sqrt{2}z)$ . Hence when we scan through the first crossover at  $l_\perp/a_F \sim -1/n(0)l_\perp \sim -\frac{l_z}{l_\perp\sqrt{N}}$ , the cloud shrinks down while preserving the Fermi-like shape. The density distribution remains unchanged until the next crossover at  $l_\perp/a_F \sim \frac{l_z}{l_\perp\sqrt{N}}$  is reached. This is the universal or unitarity limited regime as it includes the point  $a_F = \infty$ . In the region  $l_\perp/a_F > \frac{l_z}{l_\perp\sqrt{N}}$  the system gradually transforms to a weakly coupled Bose gas with a parabolic density profile  $n_B(z) = n_B(0)(1 - z^2/Z_{TF}^2)$ , where  $n_B(0) = a_B^{-1}(l_\perp Z_{TF}/l_z^2)^2$  and  $Z_{TF} = (3a_B l_z^4 N/4l_\perp^2)^{1/3}$ . The evolution of  $n(z)$  can be visualized in terms of the following two-step dependence of the mean-square size,  $\langle z^2 \rangle = N^{-1} \int z^2 n(z) dz$ , on  $l_\perp/a_F$

$$\langle z^2 \rangle = \begin{cases} \frac{1}{4} l_z^2 N, & \frac{l_\perp}{a_F} < -\frac{l_z}{l_\perp\sqrt{N}} \\ \frac{1}{8} l_z^2 N, & \frac{l_\perp}{|a_F|} < \frac{l_z}{l_\perp\sqrt{N}} \\ \frac{1}{5} l_z^2 \left( \frac{3l_z a_B N}{4l_\perp^2} \right)^{2/3}, & \frac{l_\perp}{a_F} > \frac{l_z}{l_\perp\sqrt{N}}. \end{cases} \quad (22)$$

The most characteristic feature of quasi-1D Fermi

gases is a long, well pronounced plateau with  $\langle z^2 \rangle \approx \frac{1}{8} l_z^2 N$  in the region  $l_\perp/|a_F| < \frac{l_z}{l_\perp\sqrt{N}}$  (see Eq. (22)). This should be contrasted to the plain single-crossover structure of  $\langle z^2 \rangle$  in 3D traps, which has been observed in Ref. [8]. Therefore the observation of such a plateau around the Feshbach resonance in a highly elongated trapped Fermi system unambiguously indicates a realization of the quasi-1D regime.

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[10], and  $g_B \approx 4a_F/ml_\perp^2$  for  $l_\perp/a_F \gg 1$ , which is in agreement with the 3D unrenormalized bosonic scatter-

ing length  $a_B^0 = 2a_F$  (see, for example, Ref 22).